GFD I, Problem Set 5, 2012

Due at the start of class, Wednesday 3/7/2012

The linear, inviscid, Boussinesq, *f*-plane equations for flow of a continuously-stratified fluid may be written as:

X-MOM	$u_t - fv = -\frac{1}{\rho_0} p'_x$
Y-MOM	$v_t + fu = 0$
Z-MOM	$w_t = -\frac{1}{\rho_0} p'_z + b$
MASS	$u_x + w_z = 0$
DENS	$b_t = -wN^2$

Where we have assumed $\partial/\partial y = 0$ and we have redefined the density perturbations in terms of a "buoyancy" $b \equiv -g\rho'/\rho_0$. Physically, where b is positive that means that isopycnals are pushed down a bit, and, in the course of adjusting back to a state of rest, the buoyancy force pushes up. These are the same as the equations we used in class for IGW's.

1. We know that wave-like motions (except for Kelvin waves) can only occur when the frequency is between f and N. For the case where the frequency is close to f (and $N >> f$), the flow is in the "hydrostatic, rotating" regime.

(a) What change(s) can be made to the governing equations in this limit?

(b) In the "hydrostatic, rotating" regime the frequency is given by:

$$\omega^2 = \frac{f^2 m^2 + N^2 k^2}{m^2}$$

In this limit surfaces of constant phase, and the group velocity vector, are only slightly tilted away from the horizontal, so most of the energy propagation is in the *x*-direction. What is the *x*-component of the group velocity, $C_g^x = \partial\omega/\partial k$?

(c) Re write you answer to (b) assuming that the waves were generated by moving corrugated hills with shape $z_b = \delta \sin\left[k\left(x - Ut\right)\right]$, exactly as done in class. Express your answer just in terms of U and the ratio f/ω.

(d) How does C_g^x behave as the wave frequency approaches the Coriolis frequency? Compare this result with the group velocity of Poincare waves in the same limit. What can you conclude about the effects of Earth's rotation on the ability of waves to transmit energy?

2. Now consider the other limit, where the wave frequency is closer to N, the so-called "non-hydrostatic, non-rotating" regime.

(a) What change(s) can be made to the governing equations in this limit? Is there any v velocity?

(b) Using DENS, and the solution for the vertical velocity due to flow over our hills:

$$w = -\delta k U \cos\left(kx + mz - \omega t\right),$$

Please derive the full expression for the buoyancy, b. Explain how your result makes sense near the topography (approximately at $z = 0$).